Some Hybrid Imaging Modalities

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Why so many?

X-ray CT, Emission tomography (SPECT, PET), Magnetic resonance imaging (MRI), Optical tomography (OT), Electrical impedance tomography (EIT), and others ...
Why do we need so many?
They are able to "see" different things. They are also different in terms of

- Safety
- Cost
- Contrast
- Resolution

No justice in the world!

There seem to be no cheap, safe, high contrast, and high resolution methods! 

Q: What can one do?

A: Plagiarise! (Tom Lehrer) Sorry! Combine, hybridize.

Now the **hybrid methods** come into play!

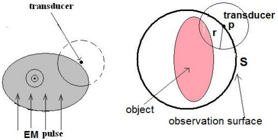
Hybrid methods

The following “classification” is “for external use only”:

- Image registration (overlap of two images obtained by different procedures) – used for a while.
- Data obtained by one imaging method used to improve the reconstruction of another, e.g. MREIT, CDI, MRE, CT-SPECT combination, ...
- In this lecture, I reserve the name **hybrid modalities** for the techniques that involve two physically different types of signals (radiations), where one (e.g., ultrasound) either modifies, or triggers another (e.g. electromagnetic) with the goal to alleviate their individual deficiencies and combine their advantages.

TAT/PAT - thermo/photo-acoustic tomography

Kruger – initiator, Agranovsky, Ambartsoumian, Ammari, Anastasio, Arridge, Bal, Burgholzer, Cox, Finch, Haltmeier, Hristova, Kang, Kowar, Kuchment, Kunyansky, Nguyen, Palamodov, Patch, Popov, Quinto, Rakesh, Scherzer, Stefanov, Sushko, Tarvainen, Uhlmann, Wang, Xu, ...



Recovering $f(x)$ from its restricted spherical means $M_S f(p, r)$. Standard issues resolved recently: uniqueness, inversion, stability, range, incomplete data effects.

The truth about TAT

If $c(x)$ - sound speed, the model is:

$$\begin{cases} \frac{\partial^2 u}{\partial t^2} = c^2(x)\Delta u, & \text{in } \mathbb{R}^3 \times \mathbb{R}_+ \\ u(0,x) = f(x), \frac{\partial u}{\partial t}(0,x) = 0 \end{cases}$$

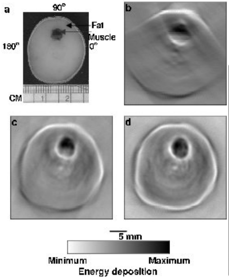
Inversion of the operator $f \mapsto g := u \,|_{S \times \mathbb{R}_+}$
Reduces to spherical means for constant sound speed only.
Related to spectral theory, transmission eigenvalues, spectral geometry, number theory

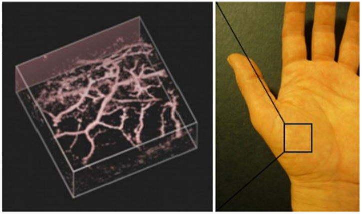
More about TAT

Reconstruction methods available:

- **FBP formulas** - available in all dimensions, stable; work only for constant speed, known for S - sphere only, do not work when f extends beyond S.
- **Eigenfunction expansions** - all dimensions, stable, (theoretically) for variable speed, S -arbitrary, work when f can extend beyond S; probably unfeasible numerically for variable speed.
- **Time reversal** - all dimensions, stable, easy to implement, variable speed, any S, any location of f.

Some TAT reconstructions





Q.: Can one recover the speed from the same data?
A.: ??? Some advances – in talks by Stefanov and Nguyen.

Quantitative photoacoustics

Arridge, Bal, Beard, Cox, Jollivet, Jugnon, Kui Ren, Tarvainen, Uhlmann, ...

The usual TAT/PAT procedure essentially recovers the energy deposition function $H(x)$. How does it relate to the actual electric or optical parameters of the medium? It is less of a problem for TAT, where radio frequencies are used.

In PAT - the radiation transport equation regime $H(x) = \sigma(x)I(x)$. Can one recover the absorption and scattering coefficients?

The diffusion regine

$$H(x) = \Gamma(x)\sigma(x)u(x)$$

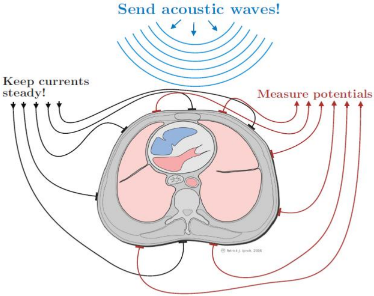
Γ - Grüuneisen coefficient, $\sigma(x)$ - absorption coefficient, $u(x)$ - radiation intensity.

$$-\nabla \cdot D(x)\nabla u(x) + \sigma(x)u(x) = 0$$

D - diffusion coefficient. Can one recover (D, σ, Γ) from $H(x)$?

Acousto Electric Tomography (AET)

Ammari, Bal, Bonnetier, Capdeboscq, Fink, Kang, Kuchment, Kunyansky, Scherzer, Steinhauer, Triki, **Wang**, Xu, ...



Focusing the ultrasound.

Synthetic focusing

Kuchment & Kunyansky , Wang at al

Use a different (unfocused) basis of ultrasound waves and focus (change basis) "synthetically."

The useful (and stable) options:

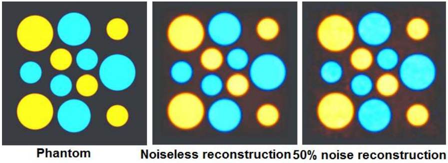
Planar transducers creating planar waves $\Rightarrow$ Fourier transform inversion.

Point transducers creating spherical waves $\Rightarrow$ TAT inversion (sic!).

Narrow US beams $\Rightarrow$ Radon transform inversion.

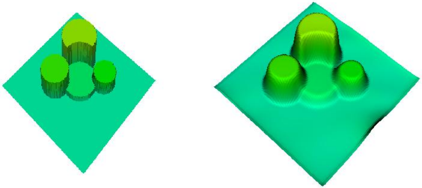
AET - reconstructions

Ammari, Bal, Bonnetier, Capdebosq, Fink, Kang, Kuchment, Kunyansky, Scherzer, Steinhauer, Wang, Xu, ...



UOT

Allmaras, Bal, Bangerth, Dobson, Kuchment, Nam, Oraevsky, Schotland, Steinhauer, Uhlmann, Wang, ...
A combination of OT with ultrasound irradiation, similarly to AET.
Use of coherent and incoherent light.
Internal information measured $G(x,d)A^2(x)I(x)$, A - ultrasound power, I light intensity, G - Green's function, d - detector position.



Absorption coefficient reconstruction (coherent light model).

Why does interior information stabilize?

A folklore meta-statement: "appropriate" internal information stabilizes the severely unstable problems like diffused OT or EIT.
Particular cases justified in the previously mentioned studies.
general question: What kind of a function $F(D(x), \sigma(x), u(x), \nabla u(x))$, if known, stabilizes the inverse boundary problems for

$$-\nabla \cdot D(x)\nabla u + \sigma u = o?$$

A rather general (brand new) answer by Steinhauer.

Magnetic resonance elastography (MRE)

Ehman, Manduca, McLaughlin

Using MRI data to recover mechanical properties of biological tissues (e.g., stiffness).

MREIT, CDI (Current density imaging)

Seo et al (MREIT), Joy, Nachman, Tamasan, Timonov ...
MRI data are used in conjunction with EIT to arrive to a stable mathematical problem and good reconstructions.

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Thanks

Thank you for your attention!

Thanks, Bill Rundell, for exceptional organization of this wonderful conference!!!

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Recent (2010 -) TAT/PAT publications (especially concerning quantitative PAT, time reversal, and speed recovery)

by Arridge et al, Bal et al, Nguyen, Stefanov, Uhlmann

See their Web pages

AET books, surveys and some papers

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UOT books, surveys and some papers

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